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THE MUTUAL ACTION OF AIRPLANE BODY AND POWER PLANT

By Martin Schrenk

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THE MUTUAL ACTION OF AIRPLANE BODY AND POWER PLANT\*

By Martin Schrenk

I. PROBLEM

In a previous report of this periodical (reference 1) the writer developed a general curve for the "power required for level flight," which is invariant against changes in airplane dimensions. It was felt that a similar general curve for the "effective thrust horsepower" was needed, which then would encompass the complete flight performance conditions.

Starting with the increase in r.p.m., a "mean thrust horsepower" curve is developed from propeller test data and engine power curves, after which this curve is then brought into relationship with the curve of the "power required for level flight" by examination of throttle flight conditions. The singular mutual relationship is manifest from the behavior of the r.p.m. This arrangement incidentally reveals a surprisingly simple relationship between the airplane dimensions primarily involved with the propeller efficiency. The effect of altitude of flight is accounted for by scale correction.

This "general power balance" of the airplane, which embraces, besides the already known quantities, only one new one, the "excess power factor," then affords the desired comprehensive survey. The profound effect of the excess power becomes evident, rate of climb, maximum speed, angle of climb and ceiling can be expressed in the excess power figure, and the revolution speeds in climb by any engine power law as well as other processes, not visible otherwise, can be examined readily. Extension to include adjustable blade propellers presents no difficulties.

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\*"Ueber das Zusammenwirken von Flugwerk und Triebwerk." From Zeitschrift für Flugtechnik und Motorluftschiffahrt, December 14, 1931, pp. 695-702; and December 28, 1931, pp. 721-727.

## II. NOTATION

In connection with the previously cited report (reference 1), the following symbols are used:

Airplane structure:

G gross weight,

$b_i$  induced span (i.e., span of equivalent monoplane with elliptic lift distribution),

$F_w$  equivalent flat plate area ( $W/q$ ) chiefly,

$F_{ws}$  total equivalent flat-plate area (connected with the dynamic pressure, hence the profile drag also); to be assumed constant by parabolic polar and suitable choice of  $b_i$ .

Power plant:

N engine power,

$\eta$  efficiency of propeller in airplane (with respect to gliding flight polar with like  $c_a$ ),

$N\eta$  thrust horsepower (utilized by propeller for propulsion of plane),

$M$  torque  
 $\omega, n$  revolutions } of engine at propeller shaft.

Propeller:

u tip speed,

D diameter,

$F_s$  propeller disk area  $\left(\frac{\pi D^2}{4}\right)$ ,

$\lambda$  coefficient of advance  $\left(= \frac{u}{v}\right)$ ,

$k_s$  effective thrust  $\left(= \frac{S}{\rho/2 F_s u^2}\right)$ ,

$k_d$  effective torque  $\left(= \frac{N}{\rho/2 F_s u^3}\right)$ ,

$c_s$  loading coefficient  $\left( = \frac{S}{\rho/2 F_s v^2} = \frac{k_s}{\lambda^2} \right)$ ,

$c_l$  performance coefficient  $\left( = \frac{N}{\rho/2 F_s v^3} = \frac{k_d}{\lambda^3} \right)$ ,

$c_d$  power coefficient  $\left( = \frac{N}{\rho/2 \pi v^5} = \frac{k_d}{\lambda^5} \right)^*$ ,

$\eta_s$  effective interference  $\left( = \frac{\eta_s}{\eta_{free}} \right)^{**}$ ,

#### Complete airplane:

$\alpha$  excess power factor, i.e., the ratio of excess power  $(N\eta)_{best}$  to  $N_{sc}$ , \*\* the power required for level flight,

$\frac{F_s}{F_{ws}}$  ratio of driving areas\*\*,

$\frac{F'_{ws}}{F_{ws}}$  ratio of interference areas, i.e., ratio of exposed interfering equivalent flat-plate area  $F'_{ws}$  to total equivalent flat-plate area  $F_{ws}$ , \*\*

#### Flight performances:

$v$  path velocity,

$\rho$  air density,

$q$  dynamic pressure  $\left( \frac{\rho}{2} v^2 \right)$ ,

$w$  rate of climb,

$w_s$  sinking speed,

$N_s$  power required for level flight,  
 $N_s = N_{sc} f(v)^{***}$ ,

$H$  altitude of flight,

\* See section IV.

\*\* See section V.

\*\*\* See reference 1.

$H_g$  ceiling (theoretical),

$H_c$  maximum altitude up to which level flight with best L/D ratio is possible.\*

Subscripts:

O sea level.

c attitude at best L/D ratio ( $v_c$ ,  $N_{sc}$ ),

best "best" operating attitude of propeller and engine,\*\*

v critical altitude (i.e., that altitude up to which the compressor is able to supply the engine with air at ground-level pressure).

### III. REVOLUTIONS-PER-MINUTE PICK-UP

#### 1. Character of Engine Power Curve

In general, the curve of the thrust horsepower  $N\eta$  is calculated by disregarding the increase in r.p.m., as if the engine operated always with constant r.p.m. and power in all flight attitudes. In this case, obviously, one single statement relative to these two quantities, suffices. But in order to follow up the effect of the variations in r.p.m. the interdependence between engine power and r.p.m. must be known.

Being a purely empirical relation, one attempts to express it by an appropriate simple mathematical approximation, that is, by power formulas.

The logarithmic plot (fig. 1) represents the full load brake curves for a number of German aircraft engines. They show totally different character. The tangents drawn with 1:1 and 1:2 slope, correspond to parabolas of the first and second order. Where the 1:1 tangent touches the curve, we have

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\*See section VI.

\*\*See section IV.

$$N \sim n \quad \text{or} \quad M = \text{constant} \quad (1a)$$

but, where 1:2 slope tangents touch, we have

$$N \sim n^{1/2} \quad \text{or} \quad M \sim n^{-1/2} \quad (1b)$$

The principal running conditions of all engines are within ambit of these two equations. From the point of view of engine technique, it is noteworthy that radial engines operate by almost constant torque, whereas the primary operating range of the 6 and 12 cylinder-in-line engines is by markedly decreasing torque.\* For the purposes of the present report, it amply suffices to use the two formulas - (1a) and (1b) - selectively as basis. This also ensures the inclusion of the intermediate parts of the engine power curves with sufficient accuracy.\*\*

## 2. Equation of r.p.m. Pick-Up

The starting point is the law of dimensions for propellers:

$$N \sim k_d \rho n^3 D^5 \quad (2)$$

So long as  $N$  and  $n$  are considered constant, so long the effective torque  $k_d$  must remain constant also. But this is not at all the case in reality. In the illustration (fig. 2)  $k_d$  generally evinces a slight increase at start, then in the range of normal flight attitude, drops slowly at first, then rapidly.\*\*\* Thus, if the power delivered by the engine did remain constant in spite of the changed r.p.m. (engine operates at maximum power), equation (2) would afford

$$n \sim k_d^{-1/3} \quad (3c)$$

\*Apparently the filling in radial engines is essentially better as a result of the uniform and short intake distances. Vibrations in the exhaust pipes may also be involved. Some radial engines use induction chambers, as the "Hornet," for instance.

\*\*For exact calculation, the exponent in (1b) can be varied as needed.

\*\*\*The amount of the rise is uncertain because the wind-tunnel data on propellers in the proximity of zero coefficient of advance are no longer reliable. The propeller then acts (in tunnels with return passage) as blower and falsifies the results. (See reference 2.)

As a matter of fact the engine power increases with the r.p.m., hence the propeller will speed up more than (3c) indicates. The measure of the increase is found by writing (1a) and (1b) for the course of  $N = f(n)$  into the fundamental propeller equation (2) as

$$n \sim k_d^{-1/2} \quad (\text{for } N \sim n) \quad (3a)$$

and  $n \sim k_d^{-1/2.5} \quad (\text{for } N \sim n^{1/2}) \quad (3b)$

Consequently, the revolutions change within the practical operating limit as the 2d to the 2.5 root of the effective torque  $k_d$ .

#### IV. THE GENERAL USEFUL POWER (OF PROPELLER) CURVE

##### 1. The "Best" Operating Attitude

In the preceding report (reference 1) the curve of the "power required for level flight" was made invariant against dimensional changes by the selection of a predetermined point of operation (that for best L/D ratio) as reference point and basing the whole curve upon it. A similar method is used for the propeller performance curves, for no insuperable obstacles intervene to find a suitable reference point. But it appears a hopeless case of deriving the course of the  $N\eta$  curve in a rational way, perhaps by resorting to a satisfactory approximation such as the parabolic polar portrays for the "power required in level flight." It lies in the character of the complicated processes on the propeller: the theory admits of a certain operating attitude with satisfactory approximation and even gives the medium for the cursory pursuance of other operating attitudes, but it is impossible to deduce therefrom a simple law for the behavior of the efficiency throughout the different attitudes:

Fortunately, there is one helping circumstance which already has quietly and unobservedly repaired untold errors and inaccuracies of propeller designers, namely, the equalizing effect of the flow on the propeller, which strives toward the particular best attitude and largely diminishes the effect of shape discrepancies on the efficiency.

Logically, the reference point is a point excellent in its importance for the operation of the propeller. Unfortunately, the point here is not quite as well defined as on the airplane structure. One might, at first thought, think of the point of maximum efficiency. (Reference 3.) But a study of a series of efficiency curves of systematically varying propellers (fig. 3) reveals forthwith that the maximum value of the efficiency of the single propeller does in no way imply the maximum efficiency which can be obtained at this point. This lies, rather, on the enveloping curve, and it is natural to select that point where the individual curve touches the enveloping curve as reference point. We shall refer to it as "best" operating point.\* But propeller efficiencies can equally well be plotted against parameters other than the coefficient of advance. Whichever one chooses depends upon the purposes in view. We have four interdependent quantities:\*\* engine power  $N$  (thrust  $S$ , respectively), revolution  $n$ , diameter  $D$ , and flying speed,  $v$ . One of these can remain free as dependent variable; it is more expedient to use  $v$  as independent, whereas the remaining two must be selected. For illustration, given  $N$  and  $n$  (as is customary), the propeller values are plotted against the "geometrical high speed," or power coefficient\*\*\*

$$c_d = \frac{k_d}{\lambda^5} = \frac{2 N \omega^2}{\rho \pi v^5}, \quad ****$$

respectively, against the reciprocal value of the 5th root of this figure, that is,  $\lambda/k_d^{1/5}$  (as Americans lately prefer to express it). (Reference 4.) (Example, fig. 4.) There is an enveloping curve here also, on which the relevant best propeller is to be found, although the points of contact of the individual curves are not the same operating attitudes as in Figure 3.

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\*The term "best" not being without a certain arbitrariness, we use the quotation marks.

\*\*Air density  $\rho$  is disregarded for the time being.

\*\*\*The author suggests the term "Drehgrad" in place of the cumbersome "geometrical high speed," based upon

$$c_l = \frac{k_d}{\lambda^3} = \text{performance coefficient,}$$

$$c_s = \frac{k_s}{\lambda^2} = \text{loading coefficient}$$

\*\*\*\*In homogeneous units.

One must therefore decide on one definite method of plotting based upon the actual running conditions. The choice falls upon representation against the coefficient of advance  $\lambda$ , being the simplest and at the same time conforming to the actual running conditions. It signifies that r.p.m. and diameter are correctly selected and the desired power absorption is obtained only by changing the shape. This variation is readily accomplished in the modern adjustable-blade metal propellers by minor changes in pitch setting. As a result thereof, we can also base the subsequent considerations upon the conventional propeller series, in which pitch is varied while retaining the blade dimensions. (See reference 5, where the principal operating attitudes are defined by means of  $\eta = \text{area.}$ )

## 2. Mean Efficiency and Effective Propeller Torque Curves

After this explanation in the preceding section, there remains the selection of the propellers upon which to apply the results. Notwithstanding much more recent reports, the most reliable data on wood propellers appear to be those by Durand and Lesley (reference 6); at any rate, they afford the best selection. Our choice is a series of small-blade propellers with suitable contour (series  $S_1$ ,  $F_2$ ,  $A_1$ ,  $P_1$ , fig. 5), and specifically those having higher pitch rates  $H/D$ :

Propeller	7	3	82	113
$H/D$	= 0.7	0.9	1.1	1.3

The  $H/D$  ratios lower than that are discarded because of their poor efficiency. The airplane designer and the engine designer must work in cooperation to the end that the propeller shaft r.p.m. permit the use of the propitious propellers of higher pitch.

Now the reference points for the reduction of the test values are revealed in Figure 3 as points of contact of the individual curves with the enveloping curve, the "best" operating points. The corresponding values carry the subscript "best." The  $\eta$  and  $k_d$  values reduced at these points are given in Figures 6 and 7. The more important  $\eta$  curve and the plotted extrapolation points appear to be in close accord, whereas the agreement for the  $k_d$  curve is less satisfactory. There is a discrepancy, but of minor importance, because  $k_d$  is merely used to

follow up the r.p.m. and appears there in the 0.4th and 0.5th power, respectively. (Equations (3a) and (3b).)\*

### 3. Mean Propeller Performance Curves and Mean r.p.m. Curves

Now the respective mean  $N\eta$  and  $n$  curve is readily computed from the mean  $\eta$  and  $k_d$  values. With (3a) and (3b), we have:

$$\left. \begin{aligned} \text{For } N \sim n: \quad \frac{n}{\eta_{\text{best}}} &= \left( \frac{k_d}{k_{d \text{ best}}} \right)^{-1/2} \\ \frac{v}{v_{\text{best}}} &= \frac{n}{n_{\text{best}}} \frac{\lambda}{\lambda_{\text{best}}} \\ \frac{N\eta}{(N\eta)_{\text{best}}} &= \frac{n}{n_{\text{best}}} \frac{\eta}{\eta_{\text{best}}} \end{aligned} \right\} \quad (4a)$$

$$\left. \begin{aligned} \text{For } N \sim n^{1/2}: \quad \frac{n}{n_{\text{best}}} &= \left( \frac{k_d}{k_{d \text{ best}}} \right)^{-1/2 \cdot 5} \\ \frac{v}{v_{\text{best}}} &= \frac{n}{n_{\text{best}}} \frac{\lambda}{\lambda_{\text{best}}} \\ \frac{N\eta}{(N\eta)_{\text{best}}} &= \left( \frac{n}{n_{\text{best}}} \right)^{1/2} \frac{\eta}{\eta_{\text{best}}} \end{aligned} \right\} \quad (4b)$$

The values computed herefrom are plotted in Figure 8. The full lines represent  $N \sim n^{1/2}$  which is primarily important. Moreover, we added the case  $n = \text{constant}$ , in correspondence with the simple calculation without r.p.m. pick-up. Conspicuously, this curve evinces practically no variation from that for  $N \sim n^{1/2}$  below  $v_{\text{best}}$ , thus apparently fully justifying the customary omission of r.p.m. pick-up in climbing calculations. Above  $v_{\text{best}}$ , in proximity of the maximum speed, the matter becomes, of course, another question, and we will find that here the role of the r.p.m. pick-up has become of primary importance.

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\*Extrapolation with another propeller series (reference 7) yielded practically the same result. However, these propellers are, in part, markedly inferior at maximum efficiency, for undiscernible reasons.

Referring to the r.p.m. curves, we find that the pick-up keeps within moderate limits ( $\pm 5$  to 7 per cent) in the practically important range ( $v/v_{\text{best}} = 0.7$  to 1.2), but increases rapidly at higher speed.

## V. COMBINED PERFORMANCE CURVES; COMPLETE PERFORMANCE DIAGRAM

### 1. Throttle Flight, Throttle Parabolas

The mutual effect of propeller and airplane body becomes apparent - for the present without mutual definition - by an examination of the throttle flight. Figure 9 shows an arbitrary curve of "power required for level flight" and an identical one for "propeller performance"; the latter to represent also the full-load curve. The question is: How is it possible to derive arbitrary throttle curves herefrom?

The law of similitude supplies here also a simple answer. We can plot curves of equal coefficient of advance. For  $\lambda = \text{constant}$ , equation (2) yields  $N \sim n$  since  $k_d$  then remains constant also, and

$$N \sim v^3 \quad (5)$$

since  $\lambda \sim \frac{v}{n} = \text{constant}$ .

Points of equal coefficient of advance thus lie on parabolas of the 3d order. Throttle curves of the propeller performance may forthwith be drawn on these throttle parabolas by plotting the fractions of the respective full performance (with like coefficient of advance) corresponding to the throttle setting.

This applies, strictly speaking, to the simple calculation without r.p.m. pick-up. But it would still be valid with r.p.m. pick-up if the throttle performance curves of the particular engine likewise were similar with respect to the  $N \sim n$  parabolas. Unfortunately, not much can be said about this because of an almost complete lack of information on throttle performance data.\* On the other

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\*Scattered experiments indicate that problems in carburetor flow and vibrations in intake and exhaust pipes are involved.

hand, the r.p.m. pick-up in the primary operating range exerts but a minor effect on the propeller performance curve, according to section IV. So the error induced by a slight change in character of the engine power curve may be disregarded.

## 2. The r.p.m. Pick-Up Defines Position of "Best" Operating Point

With the aid of this "throttle parabola" the relation between propeller performance and power required for level flight has now been established. There remains the selection of the location of the "best" operating point.

At first sight there rises the old question: propeller for "climbing," for "speed," or a compromise? Happily the decision is made easier by another stipulation bound up with the r.p.m. pick-up, namely, the propeller must not pick up so many revolutions in level flight that the engine exceeds its maximum permissible r.p.m., or else maximum speed would become impossible without damage to the engine. On the other hand, the engine is to be utilized to its fullest extent.

We consider the "best" operating attitude as the normal load attitude of the engine. With a suitable propeller, its r.p.m. at full throttle are those determined by type test. As a rule, this r.p.m. is not its permissible maximum, although it very nearly approaches it. Naturally, the conditions differ for each case, particularly on account of contingent critical r.p.m. But in general, it may be stated that a pick-up of more than 5-10 per cent is not permitted as a rule.\*

This assumption establishes an inferior limit for the "best" operating attitude. Now, inasmuch as this approaches  $v_{\max}$  quite closely, it at the same time forms the superior limit; for in view of the general airplane characteristics our aim lies in the best possible compromise between climb and speed performance. The combination of r.p.m. curve (fig. 8) and throttle diagram (fig. 9) reveals that the

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\*If more is allowed the engine is considered as not being put to full use. Even the 5 to 10 per cent are permitted only temporarily.

That far our parabolic approximation for the engine power curve is sufficiently accurate.

above boundary of the r.p.m. pick-up suggests a location of "best" operating point in the neighborhood of the throttle parabola, which passes through  $v_c$ . In order to make matters clear (and in view of advantages discussed in succeeding sections) we now definitely place the "best" operating point of the propeller performance curve on the throttle parabola through  $v_c$ , the "best" throttle parabola. In this manner we obtain the complete performance diagram (fig. 10) with an unambiguous relationship between both performances.\*

### 3. Excess Power, Excess Factor

This diagram (fig. 10) is of universal applicability. It contains only one new parameter, the "excess factor"  $\alpha$ , which is defined herewith: By excess power we usually mean the difference between the excessive and the minimum power required. But this definition is inaccurate for it fails to state that the available power is a function of the flying speed. With the throttle parabola, a better definition is assured. On the latter we know the coefficient of advance and the effective power of the propeller are constant, which simplifies matters considerably, and above all, makes them legible. In our case the throttle parabola is convenient for the "best" operating attitude, on which the power at full throttle formed, as we know, the starting point of our whole power calculation from the engine side. Obviously, the power requirement by best  $L/D$  is not the lowest possible, but a glance at Figure 10 reveals immediately that the gain still obtainable above that amounts to no more than 2 to 4 per cent, so that we may safely overlook it.\*\*

So we define the "excess factor"  $\alpha$  as the ratio of the excess power available in the "best" operating atti-

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\*Here the r.p.m. pick-up amounts to from 5 to 9 per cent at maximum speed, depending on the excess power.

It just came to my notice that H. B. Helmbold has likewise worked (since 1927) with the assumption that the best  $L/D$  of the airplane body and the principal propeller efficiency are coordinated to the flight at ceiling level and the attitude of endurance flight corresponding to it.

\*\*The slightest bump or minor control errors are just as costly, so that the "exact" calculation is really no more than self-delusion.

tude of the propeller to that required by the best L/D:

$$\alpha = \frac{(N\eta)_{\text{best}} - N_{sc}}{N_{sc}} \frac{(N\eta)_{\text{best}}}{N_{sc}} - 1 \quad (6)$$

This definition, valid for any altitude of flight, was used for plotting the power curves in Figure 10.

#### 4. Efficiency and Airplane Dimensions

We still lack one link in our chain: the absolute value of the "power required for level flight"  $N_{sc}$  is defined by the dimensions of the airplane body, that of the propeller performance by engine power  $N_{\text{best}}$  and efficiency  $\eta_{\text{best}}$ . The latter is again a complicated mathematical experimental function with speed  $v$  occupying the star role. As a result we would be forced to carry through a complete power calculation with absolute values in order to determine the propeller performance, which is the very thing we want to avoid.

However, our assumption, which couples the "best" operating attitude of the propeller to the attitude by best L/D, presents a solution.

The coefficient of advance as well as the efficiency is constant along the throttle parabola; hence the knowledge of the efficiency in level flight by best L/D suffices for the "best" operating attitude, and a relationship is readily established\* with the airplane dimensions, containing only geometrical quantities but neither speed of flight nor engine power.

The transition is completed by means of the loading coefficient

$$c_s = \frac{S}{\rho/2 v^2 F_s} \left( = \frac{k_s}{\lambda^2} \right) \quad (7)$$

where  $F_s = \text{propeller disk area} = \frac{\pi}{4} D^2$ . The loading coefficient gives a theoretical upper limit for the efficiency of the nontwisted jet. The practical limit is a cer-

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\*As proved by the author in July, 1929, in an unpublished report.

tain distance away from it, which under ordinary conditions, experiences about 11-12 per cent additional losses through twist, finite blade number and profile drag. This practical limit is illustrated in Figure 11.\*

For level flight by best  $L/D$ , the corresponding airplane drag then is:

$$W_c = S\eta_s = 1.13 G \frac{F_{ws}^{1/2}}{b_i} \quad (8)$$

Herein  $\eta_s$  is the "effective interference";\*\* it embraces all jet effects on the airplane and vice versa (see elsewhere). The corresponding flight speed is

$$v_c = 1.06 \rho^{-1/2} \left( \frac{G}{b_i F_{ws}^{1/2}} \right)^{1/2} \quad (9)$$

These two quantities, inserted for  $c_s$  in (7) yield:

$$c_{sc} = \frac{2}{\eta_s} \frac{F_{ws}}{F_s} = c_s \text{ best} \quad (10a)$$

This expression is exceedingly informative. It shows that the loading coefficient and the efficiency of the free rotating propeller defined therefrom is, in "best" operating attitude, neither dependent upon the power nor the flight speed, but almost exclusively on the "driving area ratio"  $\frac{F_s}{F_{ws}}$ ! \*\*\* The greater  $\frac{F_s}{F_{ws}}$ , the higher the attainable best efficiency. This gives us some very fundamental knowledge for the design of airplanes: When "total equivalent flat-plate area" and propeller disk area are known, it already has set a superior limit for the efficiency which cannot be exceeded!

\*Madelung's figures from reference 8, extrapolated from  $c_l$  to  $c_s$ .

\*\*Otherwise called "jet efficiency," which, however, leads to confusion with "axial efficiency" (Hoff's suggestion.) Helmbold (see footnote, page 15) calls it "installation efficiency."

\*\*\*So named because the surfaces combined into this ratio are responsible for the advance ( $F_s$ ) and retardation ( $F_{ws}$ ). It supersedes Madelung's (Luftfahrtforschung, Vol. II, No. 5) "disk area ratio," which presupposes the erroneous assumption of constant structural drag coefficient. See Helmbold (reference 9.)

But the effective interference  $\eta_s$  is likewise under the influence of the "driving area ratio," as readily proved by means of the well-known elementary approximate derivation for  $\eta_s$ . Herein it is assumed that the axial incremental velocity is evenly graded over the propeller disk area, and specifically, over three-fourths of the disk area.\* Then, according to the elementary jet theory, the ratio of dynamic pressure in the jet to the incremental velocity,  $w$  is

$$\frac{q_s}{q} = \frac{(v + w)^2}{v^2} = \frac{S}{\frac{3}{4} q F_s} + 1$$

If parts of the airplane with the effective "interfering equivalent flat-plate area"  $F'_{ws}$  are exposed to the jet, their additional drag will amount to

$$W = F'_{ws} (q_s - q) = \frac{4}{3} S \frac{F'_{ws}}{F_s}$$

This yields the (apparent) decrease in efficiency of the free rotating propeller as "interference efficiency" or "effective interference":

$$\eta_s = \frac{S - W}{S} = 1 - \frac{4}{3} \frac{F'_{ws}}{F_s} \quad (11a)$$

Consequently,  $\eta_s$  is dependent only upon the ratio  $\frac{F'_{ws}}{F_s}$ . This ratio can also be expressed in the "driving area ratio"  $\frac{F_s}{F_{ws}}$ , by writing:

$$\frac{F'_{ws}}{F_s} = \frac{F'_{ws}}{F_{ws}} : \frac{F_s}{F_{ws}}$$

$\frac{F'_{ws}}{F_{ws}}$ , the "ratio of interference areas," reveals what part of the total equivalent flat-plate area is struck by the

\*This assumption originated with Madelung (Luftfahrtforschung, Vol. II, No. 5) and has proved useful. According to more recent English researches,  $\eta_s$  is frequently less even than assumed here. Ebert (Flight Tests for Reading Airplane Polars, etc., to be published in a future D.V.L. report) arrives at a similar conclusion. See Helmbold (reference 10).

jet; with the conventional tractor type propellers it ranges between  $1/3$  and  $3/4$ . Now (11a) becomes

$$\eta_s = 1 - \frac{4}{3} \frac{F'_{ws}/F_{ws}}{F_s/F_{ws}}, \quad (11b)$$

wherefrom the intimate relationship between "ratio of driving areas" and "effective interference" becomes manifest. Now the "interference area ratio" can be discarded from (10a) so that the loading efficiency in the "best" operating attitude becomes:

$$c_{s \text{ best}} = \frac{2}{\frac{F_s}{F_{ws}} - \frac{4}{3} \frac{F'_{ws}}{F_{ws}}} \quad (10b)$$

Since the  $\frac{F_s}{F_{ws}}$  ratio is always greater than 3 in comparatively good airplanes, it predominates in (10b) also and so defines substantially, as already stated, the attainable best efficiency.

## 5. Enlargement to Variable Altitude of Flight

The result of the discussions up to now is a simple set of curves, on which, on a general curve of "power required for level flight," which is independent of the airplane dimensions, a group of propeller performance curves with parameter  $\alpha$  was plotted against the excess power factor. (Fig. 10.)

This representation with variables  $\frac{v}{v_c}$  and  $\frac{N}{N_{sc}}$  is valid for any flight altitude because  $\rho$  does not appear in the derivations. Thus, our object would have been reached but for one thing: The amount of the reference quantities varies with the altitude, so that at each altitude the axes correspond to other absolute values (relative to the critical values). As a result of which, every perception for the altitude effect is lost, aside from the fact that the computation has to be repeated for each altitude stage.

To remedy this we simply determine the reference quantities  $v_c$  and  $N_{sc}$  for one certain altitude, say, at sea level, in all flight altitudes. Then  $v/v_{oc}$  and  $N/N_{soc}$  would be on the axes, but by doing so the simplic-

ity of Figure 10 would be destroyed; in place of the one curve for the power required for level flight, we would have a whole group with altitude of flight as parameter and the propeller performance curves would become altogether obscured. For this reason we quickly make this step retrogressive again by multiplying the variables with the inversion factor of their corresponding altitude function, namely,  $(\rho/\rho_0)^{1/2}$ . The only change occurs on the axes, where we write:

$$\frac{v}{v_{0c}} \left( \frac{\rho}{\rho_0} \right)^{1/2} \quad \text{and} \quad \frac{N}{N_{0c}} \left( \frac{\rho}{\rho_0} \right)^{1/2}$$

The throttle parabolas themselves retain their validity, as becomes readily apparent. Because for  $\lambda = \text{constant}$ :

$$N \sim \rho v^3$$

$$\text{or} \quad N \rho^{1/2} \sim (v \rho^{1/2})^3 \quad (12)$$

This equation postulates, in fact, that the points of equal coefficient of advance in our diagram are still on parabolas of the 3d order.

However, in order to forego the extrapolation of the axes for the different altitudes and to bring out the processes for varying altitude of flight, the multiplication with  $(\rho/\rho_0)^{1/2}$  can be avoided by plotting the scales calculated with  $(\rho/\rho_0)^{1/2}$  for suitable altitude stages. (Fig. 12.) For easier reading of these scales, we preferred the harp-like representation. By appropriate selection of the distances the connecting lines can become straight lines, which makes the tracing very simple.

Now the sense of this last discussion is simply this: It pertains to a transformation of the whole exhibit into another scale defined by the air density. This transformation extends the validity of the one system of curves to any air density wherein, however, the effect of the flight altitude is readily ascertained by the changed scale.

However, we feel that an example would aid considerably in grasping this method of representation.

An airplane with normal engine, whose power drops somewhat quicker with altitude than the air density, has

at ground level the excess factor

$$\alpha_0 = \frac{(N\eta)_{\text{best}}}{N_{\text{soc}}} - 1.$$

In climbing flight, let us say, by constant  $L/D$  ( $\epsilon \approx \epsilon_{\text{min}}$ ) the speed and the power requirement of the airplane body increases at the rate conformably to the respective altitude. First, we compute the engine power in accordance with the given altitude performance equation.\* Since, however, the curve of the "power required for level flight" becomes independent of the height, because of our transformation, the engine power dependent on  $N_{\text{soc}}$  must again be reduced, that is, by  $(\rho/\rho_0)^{1/2}$ . This is accomplished without calculation, simply by transfer to the ordinate scale for the corresponding height. This, of course, is preceded by the multiplication of the altitude performance of the engine with  $\eta_{\text{best}}$ . With this value

we then set up the power ratio  $\frac{(N\eta)_{\text{best}}}{N_{\text{soc}}}$  and use the corresponding scale in Figure 10, where a parallel line through the abscissa at the intersection with the "best" throttle parabola yields the excess factor  $\alpha$  for the corresponding altitude. Now all flight attitudes in this height can be followed up on the respective scales along curve  $\alpha = \text{constant}$ .

## VI. APPLICATIONS

### 1. The Excess Power Factor Governs All Flight Performances

The practical aeronautical engineer is well aware of the significance of the excess power on the performance of the airplane. But this is the very first instance that formulas revealing this preeminent effect in actual figures have been set up.

The foundation is the general performance diagram (fig. 10 or 12), from which the respective rates of climb  $w/w_{\text{sc}}$  as function of the respective path velocity  $v/v_c$  with parameter  $\alpha$ , the excess factor, are taken and plotted in Figure 13. The first apparent result is the increase in path velocity for fastest climb with the excess power. Even between  $\alpha = 1$  and  $\alpha = 2$ , it already lies in

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\*For explanation, see section VI, 4, page 24.

the vicinity of the best gliding speed, which further confirms our method of making our climb calculations for best  $L/D$  instead of the attitude of minimum power required. Another fact is that it is less sensible to fly at abnormally large angles of attack for the purpose of rapid climb, the greater the excess power is at which the flight is made. For very steep climb it is a different matter. There it calls for pushing the elevator up to the utmost\* in order to command the best value. (But herein also lies the danger of this attitude when starting in restricted places!)

Now the individual performances can be found in Figure 13 as function of the excess factor and the respective basic value.

The best rate of climb (fig. 14) follows the excess factor in an almost straight line, and can be expressed in the sinking speed by the approximation formula:

$$w_{\max} \approx 0.8 \alpha w_{sc} \quad (13)$$

The best angle of climb (fig. 15) is similarly expressed by the best angle of glide:

$$\tan \phi_{\max} \approx 0.85 \alpha \epsilon_{\min} \quad (14)$$

The maximum level flight (fig. 16) passes parabola-like over  $\alpha$ . A practical and very exact mathematical approximation is

$$v_{\max} \approx (0.5 \alpha^{0.5} + 1) v_c \quad (15)$$

The respective fundamental values by best  $L/D$ ,  $\epsilon_{\min}$ ,  $v_{oc}$ , and  $w_{sc}$  are readily taken from the nomograph. (Fig. 18.)\*\*

Lastly, the ceiling is a pure function of the excess power, and may be computed along the lines laid down by

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\*The measure depends on  $c_{ac}$  :  $c_{a \max}$ . The course of the polar above  $c_{ac}$  being uncertain, the curves below  $v_c$  are shown as dotted lines.

\*\*For extension of plot, see reference 1, Figure 6.

the writer in an earlier report. (Reference 11.) There (equation (24))\* we established a relation for the ceiling by means of approximation formulas in power series for engine power and air density with altitude:

$$H_g = 11 \log \frac{w_{ho} \frac{\eta_g}{\eta_o}}{w_{so \min}} \quad (16a)$$

The numerator  $w_{ho} \frac{\eta_g}{\eta_o}$  denotes the vertical speed of ascent at ground level with the efficiency of the ceiling, or, in other words, our "best" vertical speed of ascent.\*\* The quotient in (16a) now simply is a function of the excess power, and according to our definition this equation now becomes:

$$H_c = 11 \log (\alpha + 1) \quad (16)$$

Two remarks are in order here. First, the "ceiling at best L/D,"  $H_c$  is not the "theoretical" ceiling  $H_g$  although the practical discrepancy is slight. (Reference 1.) A general rule is to figure with 100 meters loss in altitude for every 2 per cent loss of power.

Now, according to section V, at speeds below  $v_c$  with our propellers, no more than 2-4 per cent power can be attained, thus the difference between  $H_g$  and  $H_c$  amounts to no more than 100-200 meters. So  $H_c$  comes closer to the practical ceiling than  $H_g$ , and becomes about equivalent in aerodynamically poor airplanes ( $w_{sc}$  and  $\epsilon_{\min}$  great).

Besides, formula (16) is built upon a parabolic approximation for the engine power (decrease in r.p.m. inclusive)

$$\frac{N}{N_o} = \left( \frac{\rho}{\rho_o} \right)^{1.4} \quad (17a)$$

This is very convenient for the calculation, but does not satisfy the physical processes, as is readily appreciated, because the effective performance here does not

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\*Numerical factor 10.9 was later corrected to 11.0.

\*\*That is, the one to be found on the "best" throttle parabola.

disappear until  $\rho = 0$ , which is impossible on account of the friction.\* The drop in performance is at first assumed too unfavorable, and at higher altitude, too favorable.

The old Adlershof formula conforms better to actual conditions:

$$\frac{N}{N_0} = \frac{1}{\eta_m} \left( \frac{\rho}{\rho_0} - (1 - \eta_m) \right) \quad (17b)$$

Even this rectilinear law is valid only so long as the correct carburetion control is assured. (which is understood in a modern engine).\*\* Besides this drop in performance there is yet another due to the drop in r.p.m., which with fair approximation can be expressed by a minor impairment of  $\eta_m$  in (17b). As far as it is possible to conclude from the scarcity of data available, we may justly figure with  $\eta_m = 0.80$  (in 17b) as satisfactory average.\*\*\* So  $\alpha$  and  $H_c$  were defined according to this law and likewise included in Figure 17. Both curves meet at 8 km altitude; below are discrepancies up to 0.5 km, in favor of the rectilinear law.

## 2. Excess Factor and Engine Saving

The efforts to increase safety in flight and to maintain flight schedules, and in view of the still unsatisfactory operating safety of airplane engines at full throttle, have within the past few years led more and more to the opinion that pronounced throttling is imperative. (Reference 12.) In conformity with the prevalent idea, this interprets as throttling, 50 to 60 per cent full power in cruising flight. And so the question arises, just how much the airplane loses hereby with respect to its maximum speed.

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\*Compare Figure 3 of reference 11.

\*\*Unpublished tests of the D.V.L. in 1925, on a BMW-IV engine conformed to this assumption. According to an evaluation of the writer, the rectilinear law is passably correct, that is, with  $\eta_m = 0.83$  at  $n = 1500$  r.p.m. at from 0 to 6 km altitude. The loss due to drop in the r.p.m. is to be added to this.

\*\*\*The discussion, of course, applies only to engines without altitude equipment (supercharging, etc.).

Disregarding the induced drag, the 3d root of the engine power yields a superior limit for the drop in speed, which, translated for 50 per cent throttle, corresponds to about  $0.80 v_{\max}$ , and for 60 per cent throttle, to about  $0.85 v_{\max}$ . Now, the smaller the excess power, the greater the increase in induced drag effect, which admittedly increases as the reciprocal value of the dynamic pressure. Examining these conditions in the light of Figure 12 or 16, we obtain Figure 19, from which we can predict to what extent the cruising speed by varying excess power, lags behind the theoretical limit.

Airplanes which, greatly throttled, are to maintain satisfactory cruising speed, must have great excess power. For instance, when comparing two otherwise identical airplanes of different span, it may well happen that the one with smaller span has the greater speed because of its lower profile drag, whereas, in cruising with identical throttle setting the one having the larger span may be faster because of its greater excess  $\alpha$ .

### 3. Drop in r.p.m. During Climb (Unsupercharged)

In the discussion on the ceiling in a preceding section, we anticipated a result which concerns the effect of the drop in r.p.m. on the engine power during climb, and which is now examined in detail.

As before, we revert to the law of dimensions (equation (2)), and consider first the most elementary case: engine power unaffected by r.p.m. and  $k_d = \text{constant}$ . Then (2) yields:

$$N \sim \rho n^3 \quad (18)$$

$$\text{or} \quad n \sim \left( \frac{N}{\rho} \right)^{1/3} \quad (19c)$$

Accordingly, the drop in r.p.m. is forthwith amenable to solution, with the cited customary simplifications. Since  $\frac{N}{\rho} = \text{constant}$ , when  $N \sim \rho$  (indicated power), the drop is merely contingent upon the relative amount at which the effective power lags behind the indicated power, that is, of  $\eta_m$  according to altitude power. (Formula (17b).)

This simple, but special interdependence can now be amplified and supplemented. To begin with,  $k_d = \text{constant}$  no longer holds, because during climb the speed and the r.p.m. vary according to different laws. But if we retain the same throttle parabola during climb, then  $\lambda = \text{constant}$ , hence,  $k_d = \text{constant}$  also. Obviously, this applies to the conditions on the "best" throttle parabola, the starting point of our power calculation.

Conformably to (1a) and (1b), the engine power drops with the 1st and 0.5th power of the r.p.m. Consequently, the drop in r.p.m. exceeds that of (19a). Now we obtain the henceforth valid functions from (18) by division with  $n$  and  $n^{0.5}$ , respectively, thus reestablishing constancy at the left side:

$$n \sim \left(\frac{N}{\rho}\right)^{1/2} \quad (\text{for } N \sim n) \quad (19a)$$

$$n \sim \left(\frac{N}{\rho}\right)^{1/2.5} \quad (\text{for } N \sim n^{0.5}) \quad (19b)$$

With these formulas the drop in r.p.m. with altitude can be defined for any altitude power law.\* For example, we apply the law (17b) and the conventional value of  $\eta_m = 0.85$ . The results are shown in Figure 20. The influence of the altitude is greater than the effect of different r.p.m. dependency of the engine power.

To combine (19) with (17b) now would clothe the decrease in power in quite cumbersome formulas, and so we attempted to express this power decrease including the decrease in the r.p.m. by the rectilinear law. The altitude power curve computed with  $\eta_m = 0.85$  was reduced proportional to  $n$  and  $n^{0.5}$ , respectively, and plotted in Figure 20. In fact, the obtained curves are straight with very close approximation. Prolongation to the ab-

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\*The law of decrease used in N.A.C.A. Technical Memorandum No. 456 (reference 11) and extrapolated from American test flights

$$n \sim \rho^{0.12} \quad (\text{there equation (21)})$$

is subtracted here. It was

$$N \sim \rho^{1.28} \quad (\text{there equation (19)})$$

Consequently,  $\frac{N}{\rho} \sim \rho^{0.28}$  and  $n \sim \rho^{0.14}$ , respectively,  $\sim \rho^{0.11}$ , hence in close accord with  $\rho^{0.12}$ .

scissa axis yields the corresponding apparent efficiency  $\eta_m = 0.83$  and  $0.80$ .\*

It was tacitly assumed hereby, that the engine power curves remain similar with respect to the r.p.m. during altitude changes, that is, that the power maximum lies consistently at the same r.p.m., for example. The few data\*\* available on this phase of the subject appear to justify this assumption, at least for altitude up to 5-6 kilometers.

#### 4. The r.p.m. in an Altitude Engine

Elementary formulas no longer hold for supercharged engines with arbitrary altitude power course. It is true that, after  $\alpha$  has been predetermined, the requisite "best" r.p.m. is readily ascertained by the aid of the "best" throttle parabola ( $\lambda_{\text{best}} = \text{constant}$ ):

$$\eta_{\text{best}} = \eta_{0 \text{ best}} \frac{v_{\text{best}}^{***}}{v_{0 \text{ best}}} \quad (20)$$

But the calculation of  $\alpha$  presents the real obstacle, for without knowing the r.p.m., the engine power at the respective altitude cannot be determined from the assumedly given power curve; and the r.p.m. is found only through  $\alpha$ . Trial is the only remedy here, but after a little practice, results are quickly obtainable.

However, one special and quite frequently occurring case yields to direct treatment: that is, the climb with constant torque. This is the case when the supercharger supplies constant pressure up to "critical altitude" and

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\*This much more unfavorable  $\eta_m$  is well borne out in practice. From this point of view, engines with markedly decreasing torque in the principal operating attitude are preferable, because of their inferior decrease in r.p.m. and power with altitude. This reflection leads to engine with medium power to piston displacement ratio.

\*\*For example: the previously mentioned D.V.L. tests or the construction data on the Fiat engines A 20, A 22, A 25 (450, 650 and 1000 hp water-cooled 12-cylinder engines).

\*\*\*With a suitable formula the r.p.m. in throttle flight can likewise be followed up.

when its power absorption is approximately like the excess engine power during climb,\* so that the effective full engine power remains fairly constant.

For climb with  $q = \text{constant}$  and fixed blade propeller, we have, in this case,

$$\lambda = \text{constant}.$$

Moreover, it follows from  $q = \text{constant}$  ( $v/v_c = \text{constant}$ ), according to Figure 10, that

$$\alpha = \text{constant}.$$

Consequently, with  $\lambda = \text{constant}$ ,

$$n \sim v \sim \rho^{-1/2}$$

These formulas are applicable\*\* in first approximation up to a critical altitude of at least 10 km to most supercharged engines, and materially higher than that even when driven by exhaust gas turbine.

Beginning with the critical altitude the calculation can proceed as with the unsupercharged engine, at least, so long as there are no further details known about it.

### 5. Verification of Power Data \*\*\*

The deductions heretofore were primarily intended for predicting flight performances. But the relations upon which Figures 14-17 and formulas (13) to (16) are based can also be used to check the "effective" aerodynamic quantities  $b_i$  and  $F_{ws}$  as well as the propeller efficiency  $\eta$  from flown performances.

A salient feature is that one climb and one level flight suffice for computing  $b_i$  and  $F_{ws}$ , whereas the knowledge of the engine power is not necessary. This was

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\*Engine power increases through pressure difference between intake and exhaust side; e.g., see reference 13.

\*\*Additional power and blower power requirement remain about equally balanced up to this altitude.

\*\*\*In this section I had H. B. Helmbold's data and suggestions placed at my disposal.

made possible by the introduction of the excess factor  $\alpha$  as a pure function of the ceiling. (Fig. 17.) With  $\alpha$  the sinking speed and the path velocity by best  $L/D$  can be derived from Figures 14 and 16 in terms of rate of climb and maximum speed. The quotient of both is the best  $L/D$ . The nomograph (fig. 18) then yields by the agency of  $G$ ,  $v_{oc}$ , and  $\epsilon_{min}$  the values for  $b_i$  and  $F_{ws}$  in one reading.

As to the commensurate accuracy of the data, the following should be noted: quantity  $b_i$  which is decisive when predicting the climbing performance, must be proof against inaccuracies in a recheck, whereas  $F_{ws}$  with its minor effect on the climbing performances, is rather very much dependent upon the critical values and assumptions. It may be likened to a gear which speeds up in one direction and slows down in the other.

TABLE I

	$\eta_m$	$\alpha$	$w_{soe}$ (m/s)	$v_{oc}$ (km/h)	$\epsilon_{min}$	$b_i$ (m)	$F_{ws}$ (m <sup>2</sup> )
$N \sim n^{0.5}$	0.83	2.0	3.2	115	0.100	13.6	1.42
$N \sim n$	0.80	2.1	3.2	112	0.103	13.7	1.55
Source	From Fig. 20	From Fig. 17 supplemented for $\eta_m=0.83$	From Fig. 14 for $N \sim n$	From Fig. 16 for $N \sim n$	$\frac{3.6 w_{soe}}{v_{oc}}$	Nomograph Fig. 18	

TABLE I (cont'd)

	$N_{soe}$ (hp)	$N_{oc}$ (hp)	$\eta_{best}$	$\frac{(N\eta)_{horiz.}}{F_{ws}}$	$F_{ws}^*$ (m <sup>2</sup> )
$N \sim n^{0.5}$	76	111	0.685	167	1.42
$N \sim n$	76	107	0.71	167	1.47
Source	$\frac{G w_{soe}}{75}$	$\frac{N_{best}}{\alpha + 1}$	$\frac{N_{soe}}{N_{oc}}$	Ref. 11 Fig. 7	$\frac{346}{167} \eta_{best}$

The propeller efficiency is expressed as the quotient of power required for level flight by best  $L/D$  ( $N_{soc} = \frac{G w_{soc}}{75}$ ) and the to-be-absorbed engine power ( $N_{oc} = \frac{N_o \text{ best}}{\alpha + 1}$ ). Since  $\alpha$  and  $w_s$  are subject to the aforementioned conditions their inherent sources of error are obviously transferred to  $\eta$  also. A certain control check is afforded by direct calculation of  $F_{ws}/\eta$  from maximum speed and respective engine power. (Reference 11.)

These arguments are to be illustrated on a two-seat sport biplane. Carefully determined and reduced to international standard altitude, it exhibits the following characteristics:

$$H_g = 6.05 \text{ km} (H_c = 5.85 \text{ km}),$$

$$w_o = 5.25 \text{ m/s},$$

$$v_{\max.} = 205 \text{ km/h},$$

$$N_{\text{full}} = 328 \text{ hp (climb)},$$

$$346 \text{ hp (level flight)},$$

$$G = 1785 \text{ kg}.$$

The requisite engine power in "best" operating attitude for calculating  $N_{\text{best}}$  is obtained by estimating the effect of the r.p.m. from Figure 12 with a temporarily assumed  $\alpha = 2$  at  $N_{\text{best}} = 333 \text{ hp}$ ; further calculation is contained in Table I. The r.p.m. dependence of the engine power and through it its altitude dependence were varied so as to bring out the effect of these assumptions on the evaluation.

As expected, the value of  $F_{ws}$  fluctuates considerably with the engine power law, whereas  $b_i$ , the quantity controlling the climb, remains fairly constant. Comparison with  $F_{ws}^*$  defined directly from level flight reveals the correctness of the order of magnitude and the true value probably between 1.42 and 1.47. If this comparison, on the other hand, resulted in widely disparate values, it would be suggestive of unsatisfactory climbing flight, wrong evaluation of engine power, or some other error.

For a span of 12.6 m, Prandtl's biplane factor  $\kappa$  with  $b_i = 13.6$  m, amounts to

$$\kappa = \frac{W_D}{W_E} = \left( \frac{12.5}{13.6} \right)^2 = 0.85.$$

The theory would yield  $\kappa = 0.78$ ; the figure 0.85 contains, besides the other induced losses, the presumable increment of  $c_{wp}$  and  $c_{wr}$  with  $c_a$ , which, according to our method, is counted in with the induced losses.

Lastly, in accord with section V, 4, the efficiency can be divided into its elements  $\eta_{free}$  and  $\eta_s$ , as carried out for the values computed in the first line (at  $N \sim n^{0.5}$ ). With a propeller disk area of  $7.0 \text{ m}^2$ , we have:

TABLE II

$F_s/F_{ws}$	$c_s$ best	$\eta_{free}$	$\eta_s$	$F_{ws}'/F_{ws}$
4.95	0.44	0.795	0.86	0.56
Source	For $\frac{F_{ws}'}{F_{ws}} = 0.5$ from (10b)	From Fig. 11	$\frac{\eta_{best}}{\eta_{free}}$	From (11b)

The interference area ratio  $F_{ws}'/F_{ws}$  is plausible in its amount. Obviously, it is quite sensitive against minor errors in  $\eta_s$ , so that in this case also any accurate determination is contingent upon the execution of different test series, unrelated to one another, and comparison of the results. It was not the aim in this section to go into details regarding it, but merely to illustrate the multiple applicability of our method on an actual example.

## 6. Adjustable Blade Propeller and Unsupercharged Engine

This report would not be complete without including the adjustable blade propeller. It is true, that the structural problem has not been satisfactorily solved as yet in spite of many attempts. We still await a perfectly reliable, universally applicable adjustable blade pro-

propeller, which is simple to maintain, of light weight, and reasonable in price. On the other hand, there has never been a time when urgent demand did not sooner or later bring forth a solution. Experiments relative to the advantages of such propellers may provide a stimulus, whereas the lack of space restricts our discussion to a brief explanation and a few examples.

Again we proceed from model tests. Figure 21 represents one of the latest American test series on an adjustable blade propeller. (Reference 14.) The separate efficiency curves carry as parameter the angle of setting at 60 per cent tip radius; at 20.4° setting the propeller shows a constant geometric pitch of 0.7 D. It is seen that the radially increasing pitch at large blade angles affects the maximum efficiency very little notwithstanding the marked variation from the theoretical course.\*

The essential feature of the adjustable blade propeller is the freedom in the selection of the r.p.m., obtained independent of the coefficient of advance (within certain limits) and from the degree of throttling. The r.p.m. depends on the effective torque  $k_d$ . This is shown in Figure 22, not as dependent variable but as parameter of a group of curves whose individual points lie on the efficiency curve.

The r.p.m. being free, the calculation of the propeller performance curve makes necessary an assumption as to the course of the r.p.m. The simplest and at the same time, most fundamental premise is: constant r.p.m. over the entire speed range. For the calculation we simply select a suitable  $k_d$ , after which the  $k_d, \lambda$  curve forthwith reveals the power course against the speed, whereby  $k_d = \text{constant}$  is obtained by adjusting the pitch to constant r.p.m. Figure 23 shows the gain attainable in climbing flight, which here amounts to about 1/3 on account of the increase in the r.p.m., in accord with the assumed engine power law ( $N \sim n^{1/2}$ ); 2/3 are due to the better curve of the  $\eta$  values at low  $\lambda$ . (Compare figs. 3 and 21.) But at high speed the adjustable blade type appears to be perceptibly inferior.

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\*The propeller has the same maximum efficiency as aforementioned series of N.A.C.A. Technical Report No. 141 (reference 6). Obviously, being a small-blade metal propeller, it might come slightly higher under otherwise identical conditions.

Of course, this comparison is not conclusive. That is attributable to the definition of "best" operating attitude, which was temporarily retained to facilitate a transfer of the power curve onto the plot. (Fig. 12.) But in reality this concept no longer holds true because of the freedom of the pick-up in r.p.m.; for that reason a power comparison is better compared to like power at high speed, that is, to a 5-10 per cent super r.p.m. with respect to the systematically defined r.p.m. In this case the adjustable blade propeller presents even better advantages. Figure 23 affords an example for  $n = 1.08 n_{\text{best}}$ , which is equivalent to the r.p.m. of a fixed blade propeller in level flight with  $\alpha \approx 4$ .

For engines with constant torque ( $N \sim n$ ), the adjustable blade propeller appears to be even more advantageous, because here the decrease in r.p.m. of fixed blade propellers is greater and its effect on the engine power, in addition, more sensitive. Referring to Figures 8 and 23, it may be stated that the gain in thrust horsepower during climb, attainable on these premises, exceeds 25 per cent under certain circumstances; this means an increase in rate of climb of at least 30 per cent.

#### 7. Adjustable Blade Propeller and Supercharged Engine

Here is where the gain promises to be greatest. Whereas, with the fixed blade propeller the r.p.m. in climbing flight increases with  $\rho^{-1/2}$ , so that a propeller correctly dimensioned for the operating altitude supplies only a fraction of the full r.p.m. of the engine at ground level,\* here the r.p.m. can be kept constant, irrespective of the flight altitude, thus assuring the use of the full engine power at any altitude.

The assumption  $n = \text{constant}$ , according to (2) leads to the condition

$$k_d \sim 1/\rho \quad (21)$$

Besides,  $\lambda \sim v$ , hence

$$\lambda \sim \rho^{-1/2} \quad (22)$$

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\*It also denotes an extremely unfavorable working condition for the engine.

Thus, given the total  $k_d$  range of an adjustable blade propeller from model experiments (fig. 22), one can according to (21) and (22) define the  $\lambda$  for each altitude stage and the respective  $k_d$  and plot them into the  $(\lambda, \eta)$  diagram. Figure 24 shows the coefficients of advance and the efficiencies with altitude for our example with a critical altitude of 12 km. At low altitude the propeller obviously, does not operate in its best range; the operating curve, however, approaches the enveloping curve more and more as the altitude increases. In accordance with that the propeller performance curve in Figure 25 is incomparably more favorable than the fixed blade propeller. The gain in rate of climb at low altitude is very considerable. In our example, which represents average conditions, it amounts to 140 per cent. The time of climb to critical altitude is lowered by 45 per cent!

There are two other advantages of the adjustable blade propeller, namely, the possibility of markedly improving the angle of glide by adjusting the blades to the direction of the wind; conversely, it can be considerably lowered for landing and taxiing in restricted space by resetting to negative blade angle and for negative thrust. The engine, which otherwise represents only dead weight at landing, thus becomes an extremely effective means of deceleration.

## VII. SUMMARY

The present report treats of the development of general propeller performance and r.p.m. curves which, combined with the general curve of the power required for level flight, presents a complete picture of the performance. It should prove very convenient for answering many difficult problems of the airplane designer, quickly and legibly.

It is in the nature of such discoveries not to be applicable to all imaginable cases. Thus, as our curve of the power required for level flight is only an approximation for constant profile drag coefficient, hence not forthwith suitable to unconventional wing sections, so the propeller performance curve must also be handled with a certain caution in cases of propellers of abnormal blade forms or pitch ratios. For checking the performance of already designed airplanes the methodical way may occa-

sionally yield more accurate results in so far as the aerodynamic and engine data are sufficiently reliable.

But in any other case, particularly for project or design purposes or for evaluation of aircraft, the method propounded here will prove perfectly accurate. Moreover, too much refinement serves no useful purpose. Its importance lies in making separate calculations superfluous and supplying the design engineer a survey on what may be obtained, and which he cannot obtain as quickly in any other way.

The salient features and conclusions are briefly repeated, as follows:

1. The increase in r.p.m. is of secondary importance in flight performance calculations. Nevertheless, there are perceptible differences according to the character of the engine power curve.

2. The increase in r.p.m. at maximum speed predominates the selection of the propeller, so far as it does not concern purely "starting" and "climbing" propellers.

3. On a "throttle parabola" coefficient of advance and efficiency are constant, thus the r.p.m. proportionate to the flight speed. The thrust horsepower curves are built up on the throttle parabolas.

4. The "best" operating point of the "compromise propeller" shall coincide in level flight with the point for best  $L/D$ . Then the "best" throttle parabola passes through the point of best  $L/D$ . On this lie the "best" operating points of the thrust horsepower curves; they are the backbone of the whole system of curves.

5. The "best" efficiency of the free propeller is entirely dependent upon the ratio of the driving areas and (subordinately) the ratio of the interference areas. These two quantities likewise define the effective interference. Weight, span, engine power and speed are ignored.

6. The excess power is measured on the "best" throttle parabola and expressed in excess power factor. It governs all flight performances. Simple approximation formulas facilitate design and check.

7. The path velocity for fastest climb increases with the excess power; under ordinary conditions it ranges at or above the best gliding speed.

8. Improvement in aerodynamic quality (expressed by  $c_{min}$ ) by constant excess power impairs angle and rate of climb. This applies, in particular, to starting.

9. The greater the excess factor the smaller the loss in speed during cruising by prescribed throttle setting.

10. In the unsupercharged engines the excess power determines the ceiling. The effect of the altitude performance law decreases (by carburetion control) and its significance is confined to high ceiling.

11. The drop in r.p.m. in the climbing flight with an unsupercharged engine is a measure of the lag of the effective power behind the (theoretical) indicated power. Its effect can be expressed by rectilinear law as a reduction of from 2-5 per cent in mechanical efficiency, depending upon the character of the engine-power curve.

12. The adjustable blade propeller presents a perceptible gain in climbing power for unsupercharged engines, and distinct superiority in climbing for supercharged engines.

Translation by J. Vanier,  
National Advisory Committee  
for Aeronautics.

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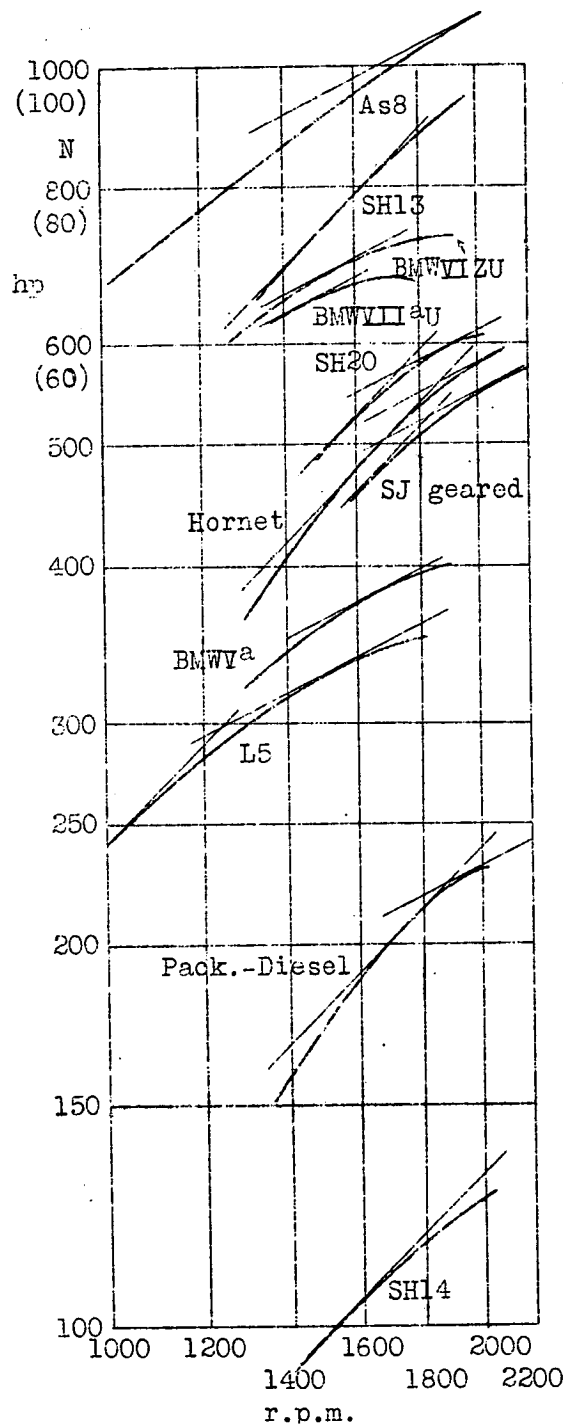


Fig. 1 Full power curves of various aircraft engines

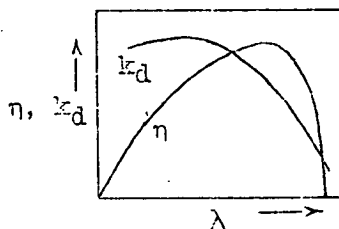


Fig. 2 Example of an  $\eta$  and  $k_d$  curve

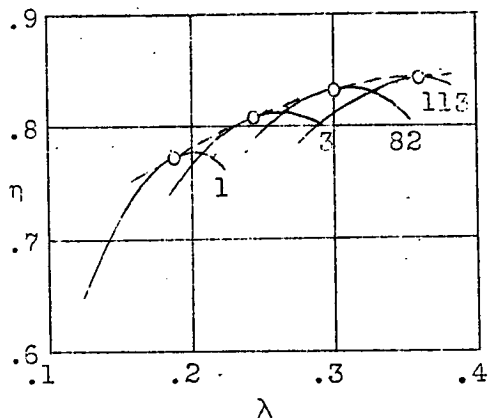


Fig. 3 Efficiency plotted against propulsive efficiency, taken from N.A.C.A. Report No. 141, propeller family having like contour but different pitch.

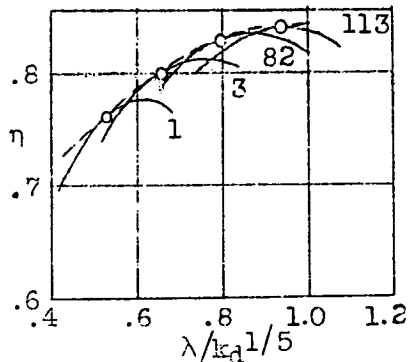


Fig. 4 Efficiency against power coefficient (series of Fig. 3)

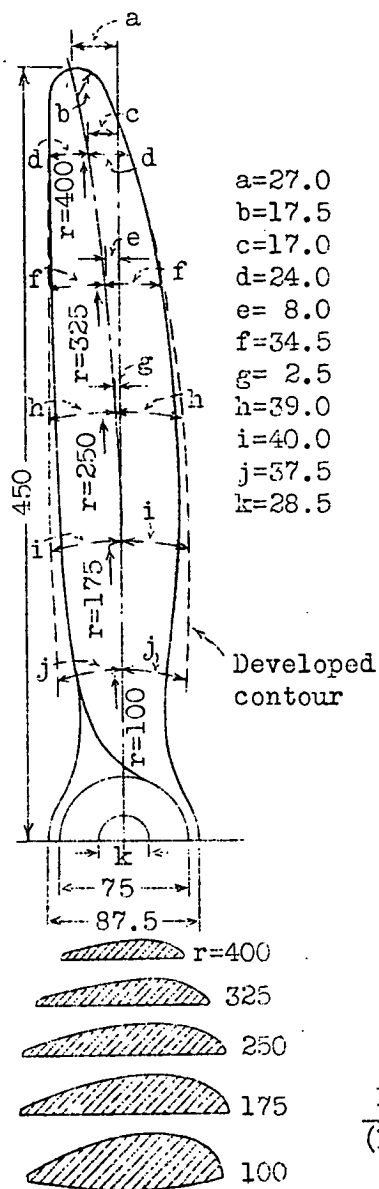


Fig. 5 Plan form of propeller series of Figs. 3,4.

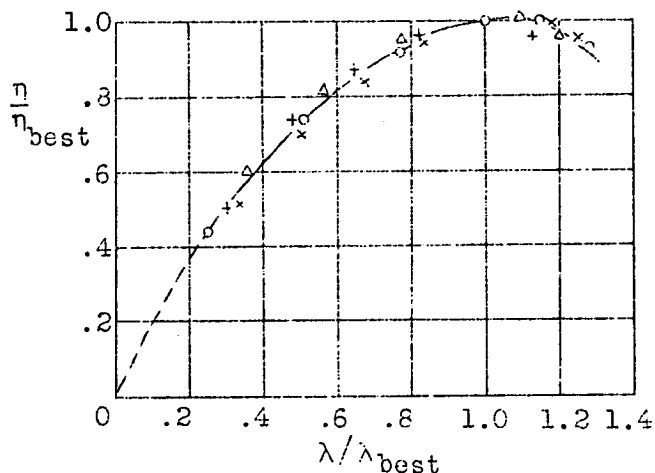


Fig. 6 Mean efficiency curve

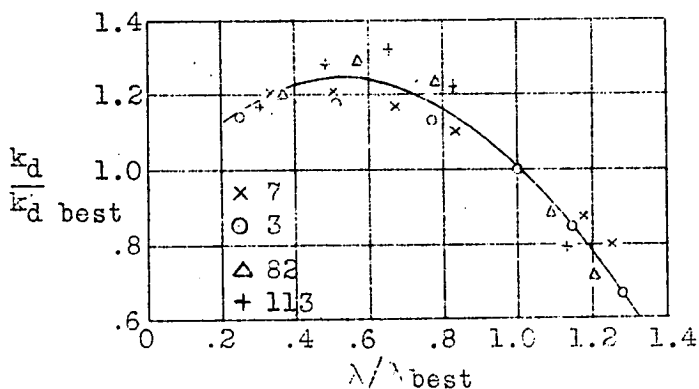


Fig. 7 Mean effective torque, the (114 points are readings from Rpt./)

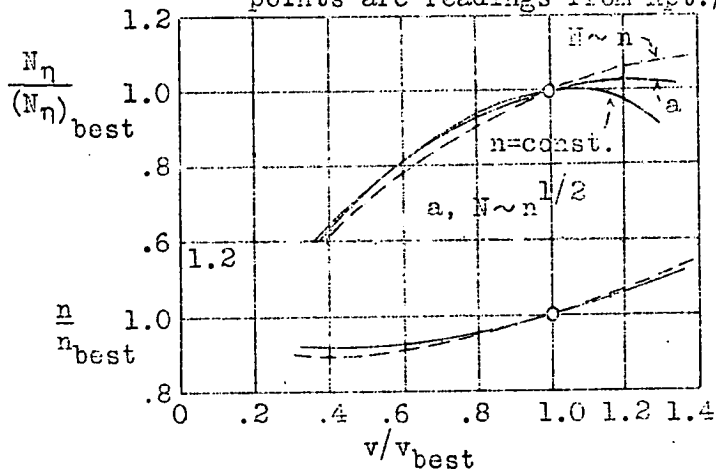


Fig. 8 Mean curves of propeller efficiency and r.p.m.

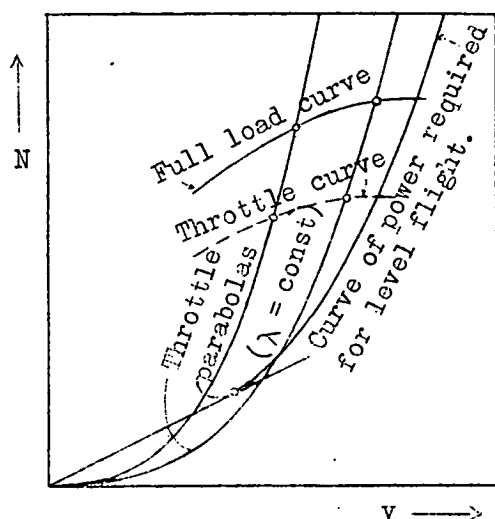


Fig. 9 Definition of power curves.

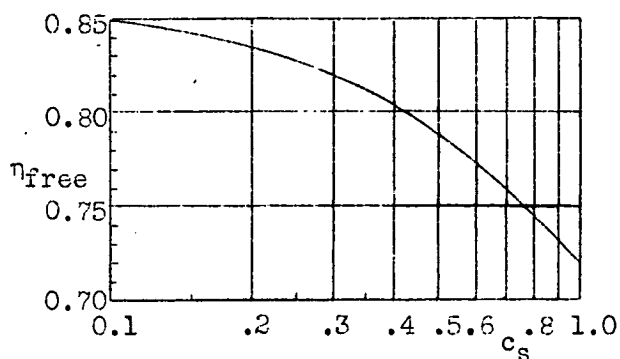


Fig. 11 Efficiency plotted against loading coefficient.

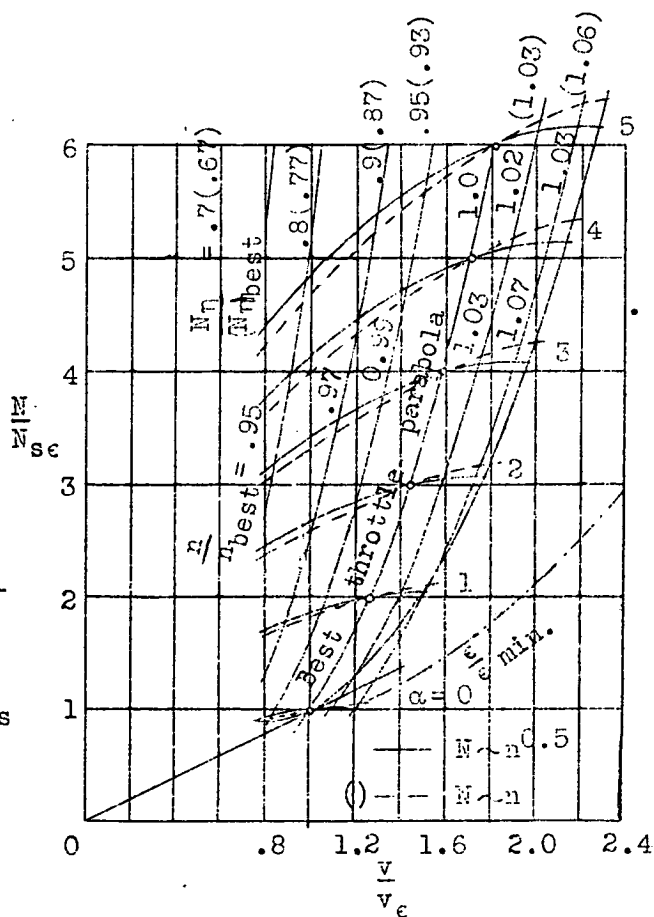


Fig. 10 General power diagram for definite flight altitudes.

The backbone of the propeller performance curves is the "best" throttle parabola.

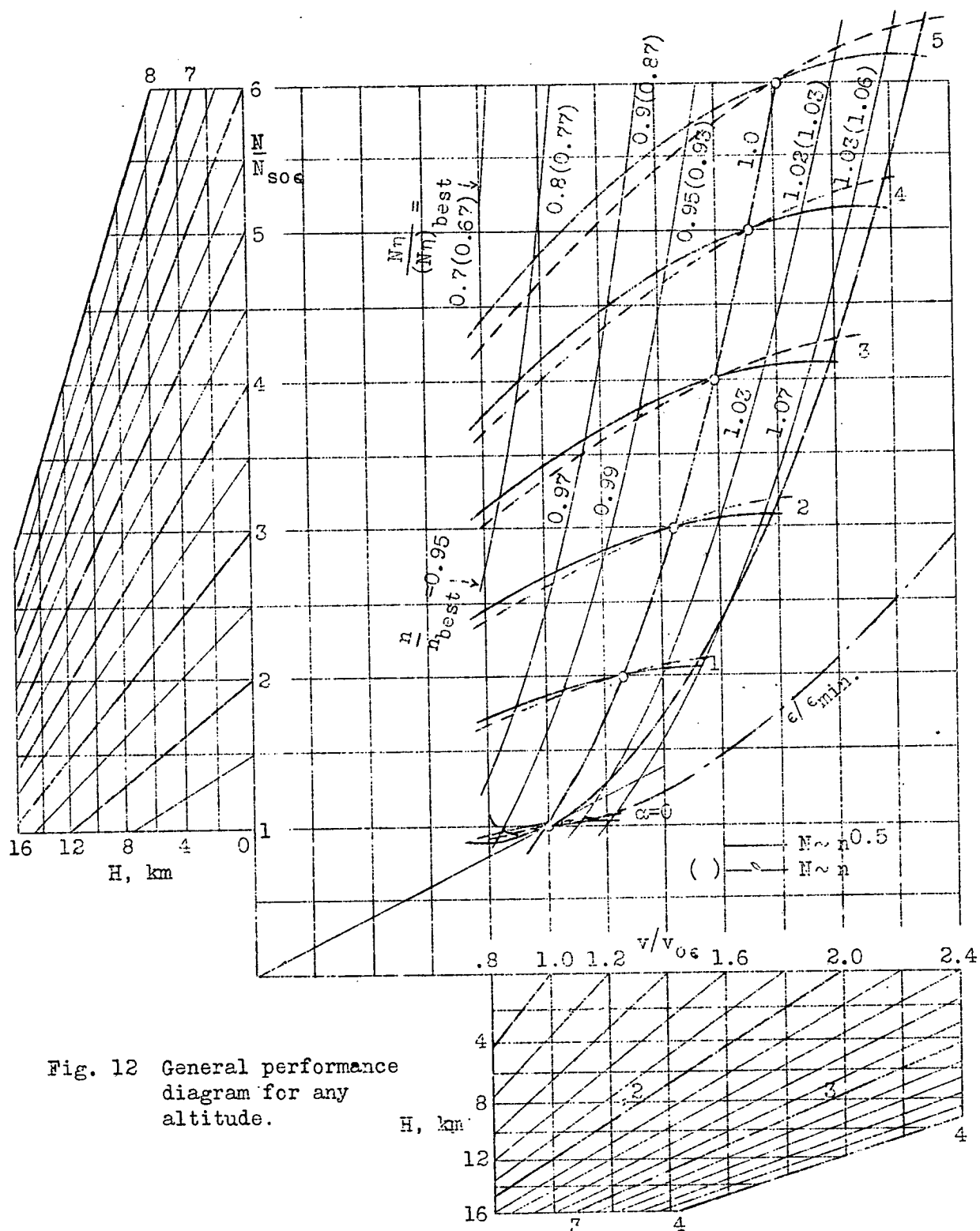


Fig. 12 General performance diagram for any altitude.

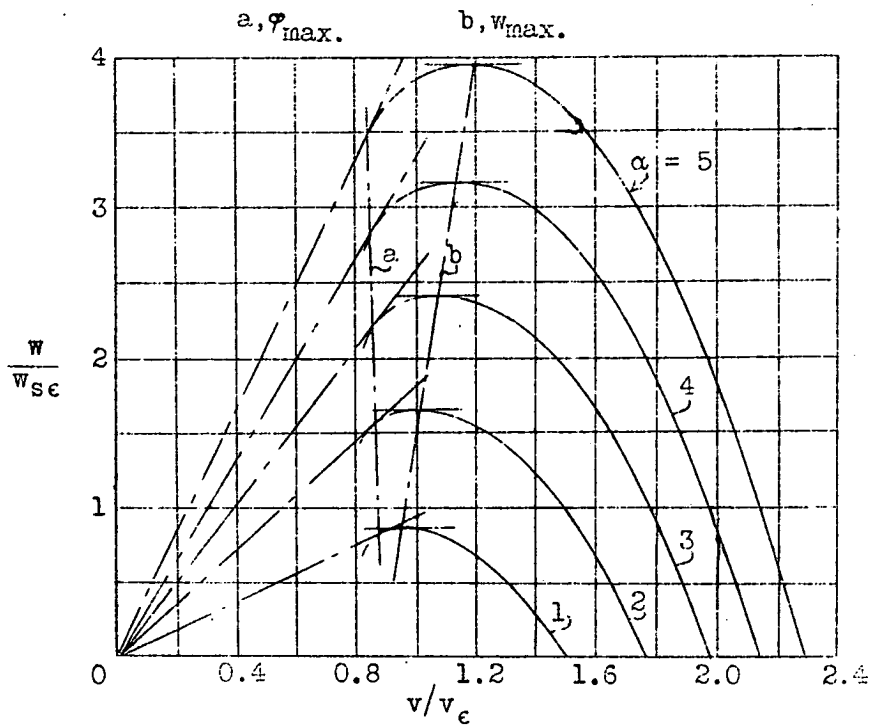


Fig.13 Rate of climb plotted against speed of flight.

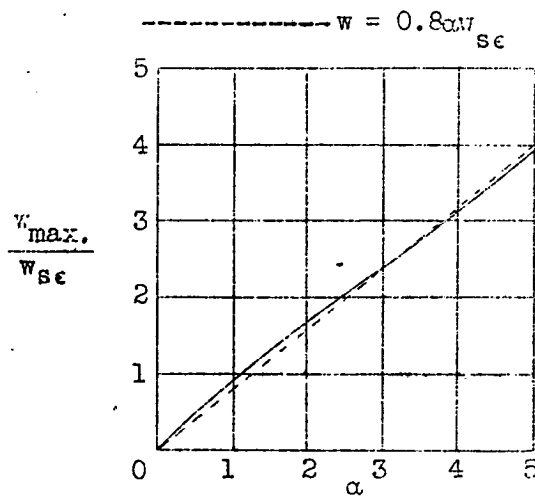


Fig.14 Maximum rate of climb against excess power.

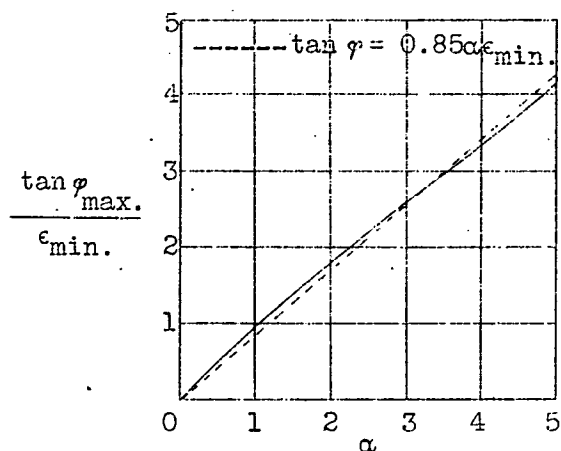


Fig.15 Maximum angle of climb against excess power.

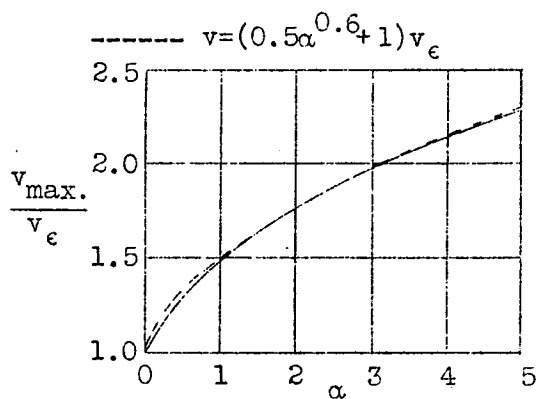


Fig.16 Maximum level flight against excess power.

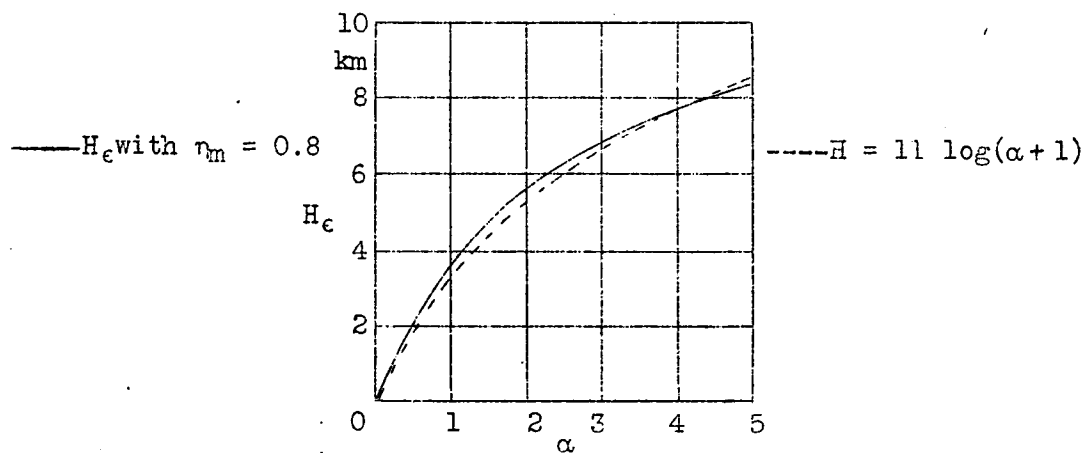
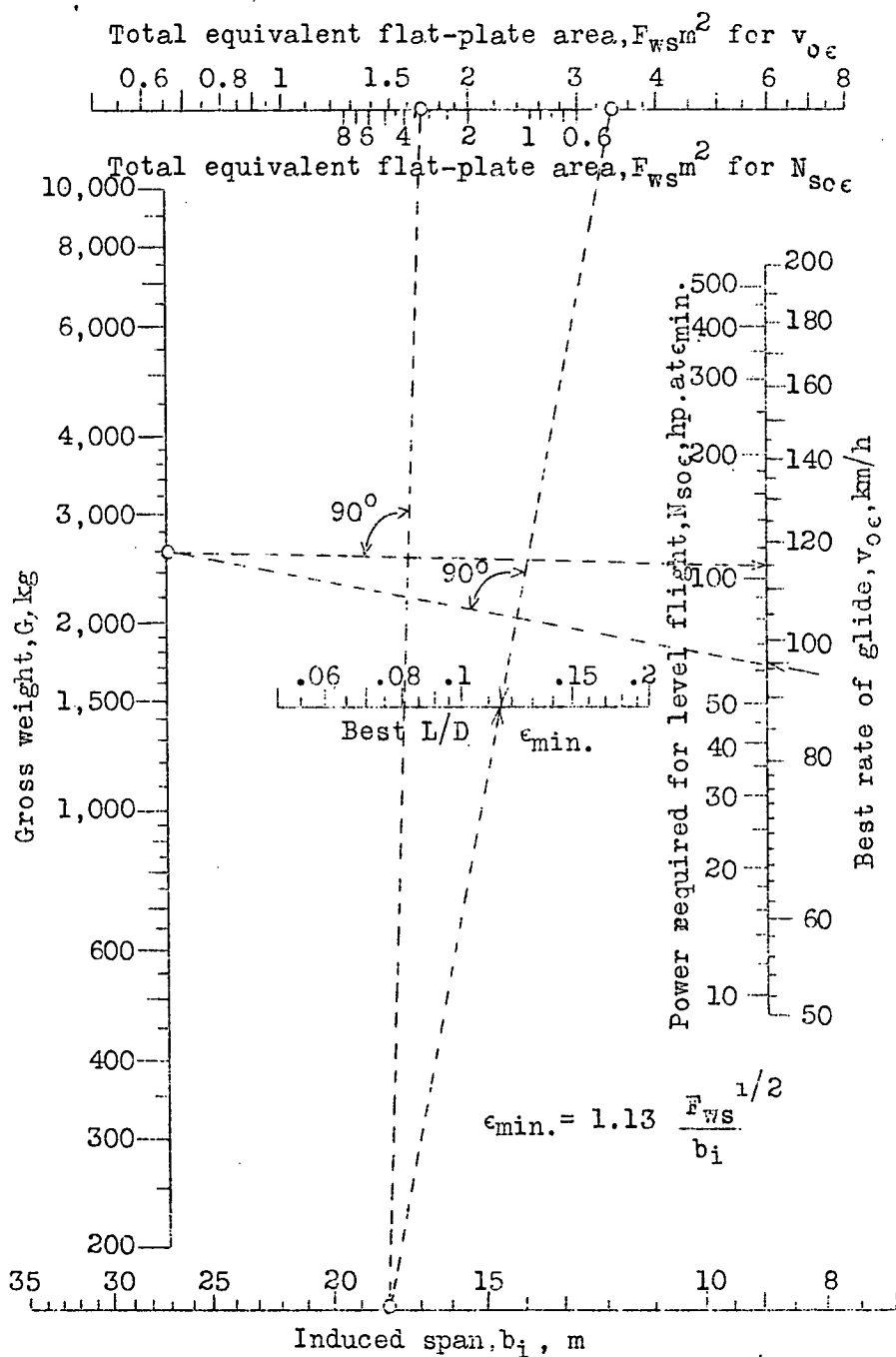


Fig.17 Ceiling against excess power.



$$v_{oe} = \frac{3.82}{\rho_0^{1/2}} \left( \frac{G}{b_i F_{ws}^{1/2}} \right)^{1/2} \quad N_{soc} = \frac{1}{62.5 \rho_0^{1/2}} \left( \frac{G}{b_i} \right)^{3/2} F_{ws}^{1/4}$$

Fig.18 Nomograph for best L/D, corresponding speed and power required for level flight. Read with celluloid sheet having a rectangular system of lines.

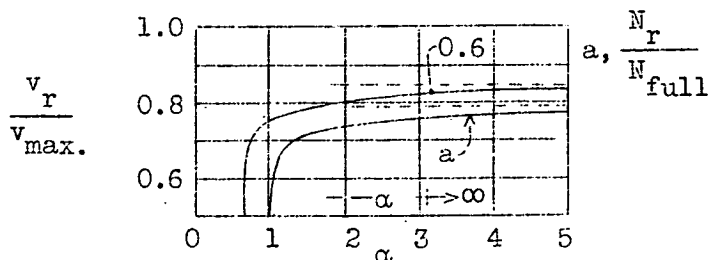


Fig.19 Cruising speed plotted against excess factor. The smaller the excess factor  $\alpha$  the more the cruising speed  $v_r$  lags behind the maximum speed for a given throttle setting  $N_r/N_{full}$ .

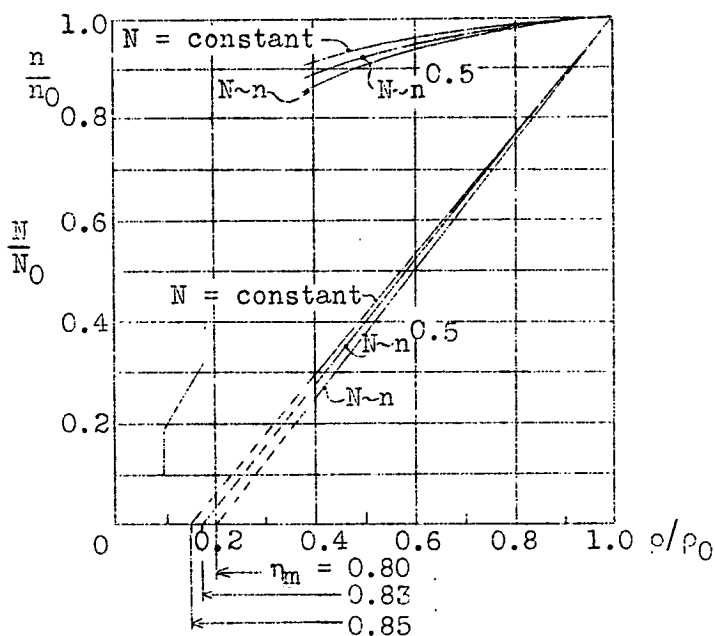


Fig.20 Drop in r.p.m. with altitude and its effect on the drop in power.

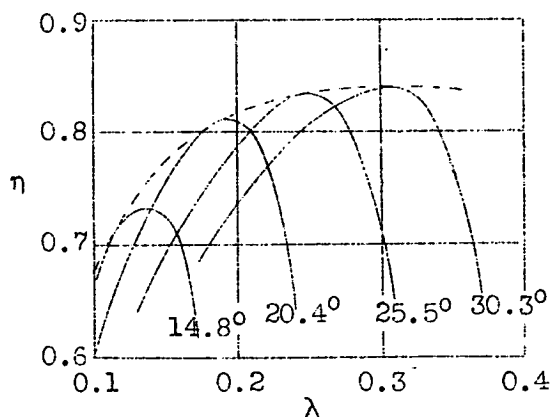


Fig.21 Efficiency curve of an adjustable blade propeller according to N.A.C.A. Technical Note No.333.

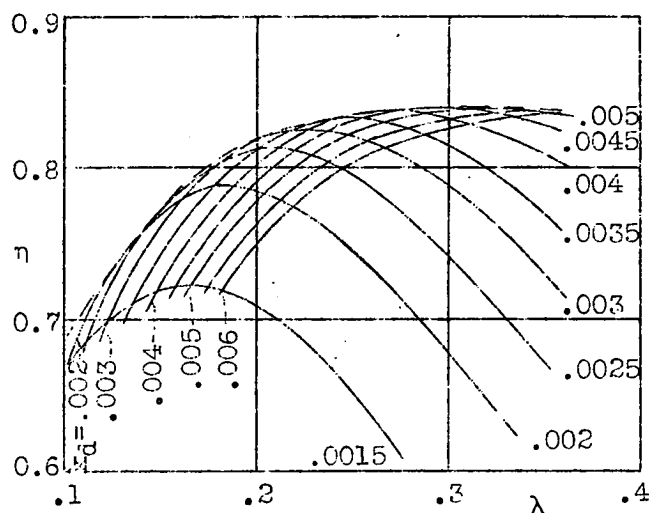


Fig. 22 Effective torque curves for the same adjustable blade propeller. This representation eliminates the measured propeller settings and permits calculation of the r.p.m. conditions.

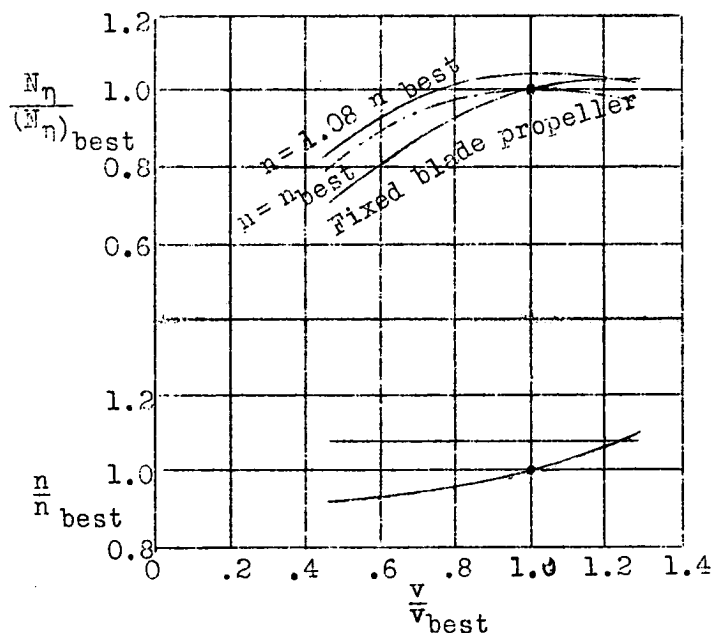


Fig. 23 Performance and r.p.m. of adjustable blade propeller compared to fixed blade propeller.

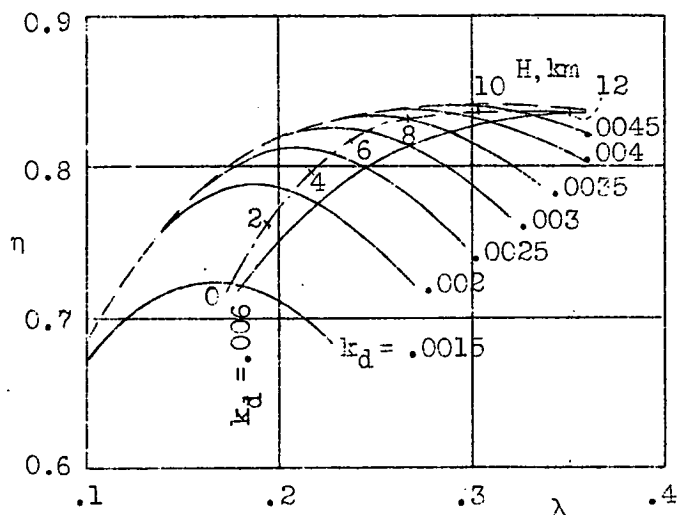


Fig. 24 Efficiencies during climb with constant power and r.p.m. By constant power and r.p.m. the effective torque is converse to the air density.

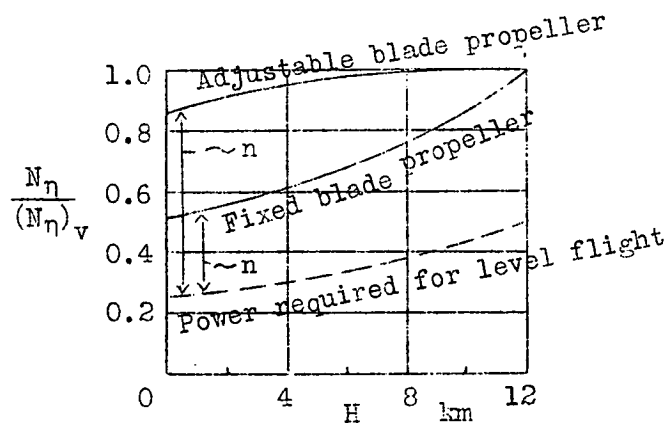


Fig. 25 Gain in rate of climb with adjustable blade propeller, 45 per cent for  $a=1$  at critical altitude.